

Collapsing open isotropic universe generated by nonminimally coupled scalar field.

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Abstract

We investigated the behavior of an open isotropic universe generated by a scalar field which couples with background curvature nonminimally with the coupling constant ξ . In particular we focus on the situation where the initial value for the scalar field ϕ_{in} is greater than the critical value $\hat{\phi}_c = m_p / \sqrt{8\pi\xi(1-6\xi)}$. The behavior is similar to an open de Sitter universe with $k = -1$ with a negative cosmological constant $\Lambda < 0$. It is found that the universe will collapse eventually to a singularity and thus has a finite extent in time in the future. Furthermore, there are some cases which shows a rebouncing behavior before the final collapse.

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1 INTRODUCTION

There is a growing interest in the scalar field in the cosmological situations because of its important role played in the inflationary universe scenario[1,2]. There the expansion of the universe is totally governed by the behavior of the scalar field. In this paper we would like to make a comment on the behavior of the universe dominated by the scalar field from the point of view so far not paid much attention. Namely we shall be interested in the universe with nonminimally coupled scalar field ϕ in some special circumstances which will be explained in short. The nonminimal coupling is described by the form $\frac{1}{2}\xi R\phi^2$ in the lagrangian where R is the spacetime curvature and ξ the coupling constant. Minimal coupling has $\xi = 0$. We choose the convention that the conformal invariance yields $\xi = \frac{1}{6}$. Particle physics do not specify any

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particular value for the coupling constant ξ , so there is no a priori reason to restrict our attention to a particular value for the coupling.

There have been many studies on the effect of this coupling on the expansion behaviour and on the chaotic inflationary scenario. It has been shown that the original chaotic scenario[3] does work only for a limited range of the coupling constant such as $|\xi| < 10^{-3}$ [5]. Fakir and Unruh[8] showed a possibility to have a successful scenario of chaotic inflation with a large negative coupling constant. On the other hand Futamase and Maeda[5] have pointed out that there will be two critical values $\phi_c = mp/\sqrt{8\pi\xi}$ and $\hat{\phi}_c = mp/\sqrt{8\pi\xi(1-6\xi)}$ for the scalar field in the range $0 < \xi < 1/6$ and there is no isotropic solution with flat and closed spatial curvature if the scalar field is larger than $\hat{\phi}_c$. Moreover Starobinskiy[7] and Futamase et al[5,6] have shown that the anisotropic shear diverges as the scalar field approaches to $\phi_c = mp/\sqrt{8\pi\xi}$ for almost all initial configurations $\phi > \phi_c$ and thus the closed as well as flat universe starting from such initial configurations does not lead to our present isotropic universe. Therefore no serious attention has been paid for the situation with such initial configurations for the scalar field.

However some of the recent observations do suggest the possibility of open universe[4] [19] and there are also studies to explore the possibility to have open inflationary scenario in theoretical side[11,17,12]. In this situation we think it may be interesting to consider the case of open universe with nonminimally coupled scalar field under the condition $\phi > \phi_c$ because the possibility to have an isotropic expansion in this situations is not excluded and thus there will be no a priori reason to reject such a situation.

It turns out that such a universe has remarkably rich behavior such that the universe recollapses to a singularity in a finite time. In this sense the effect of nonminimal coupling constant is similar with that of negative cosmological constant. More than that there are situations where the universe show rebouncing behavior before the final collapse.

This paper is organized as follows. In Sec. 2 we give general discussions including a short review about the de Sitter universe for reader's convenience and about nonminimal coupling. In Sec. 3 we turn our attention to the case of open universe ($k = -1$). There we will find some very interesting behavior of such a universe. Finally summary and remarks are give in Sec. 4.

2 General consideration

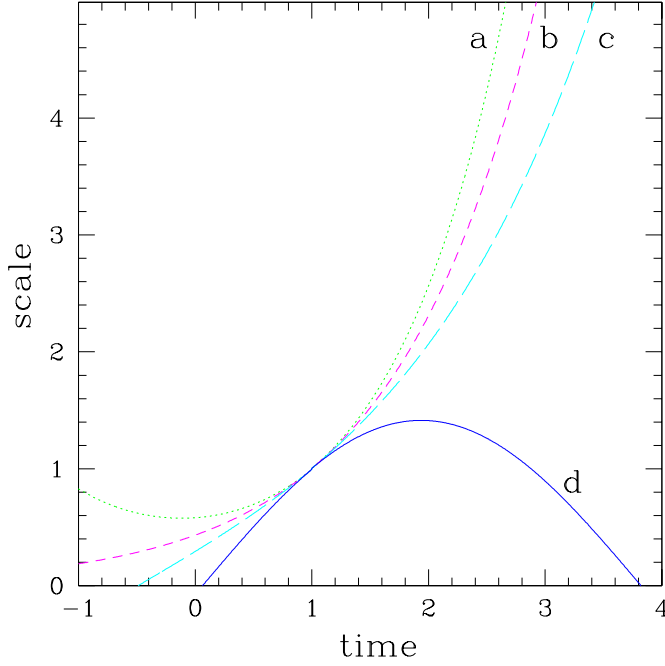


Fig. 1. The typical behavior of cosmological scale-factor in the de Sitter universe.

2.1 de Sitter universe

We briefly review the expansion behavior of de Sitter universe. In this model the cosmological constant is assumed to be $\Lambda \neq 0$ without matter. The space-time is described to be the Friedmann-Robertson-Walker (FRW) type:

$$ds^2 = -dt^2 + a^2(t) d\Omega_3^2(k) \quad (1)$$

where $d\Omega_3^2(k)$ is the metric of the universe depending on the curvature constant $k = +1, 0$ or -1 , respectively. The Einstein equations is

$$H^2 + \frac{k}{a^2} = \frac{\Lambda}{3} \quad (2)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble expansion rate.

Fig. 1, this diagram shows the typical behavior of cosmological scale-factor in the de Sitter universe. Those behavior are apparently different in the case of $\Lambda > 0$ and $\Lambda < 0$.

The lines (labeled a, b, c) are curves in the case of cosmological constant $\Lambda > 0$ with curvature $k > 0, k = 0$ and $k < 0$, respectively. In the inflationary

scenario, the vacuum energy of the scalar field (or inflaton) ϕ plays the same role with the cosmological constant $\Lambda(>0)$.

The solid line (the lowest line labeled d) is curve in the case of $\Lambda < 0$ with $k < 0$, which is the only case of solution in de Sitter universe with $\Lambda < 0$. Here the universe necessarily recollapses and thus the spacetime has a finite extent in time in the future. Those are limited by the value of cosmological constant $|\Lambda|$. Though there are some interesting features in this case, it is usually regarded as no physical meaning.

2.2 Non-minimal coupling constant ξ

We consider the total action of Einstein gravity and a real scalar field ϕ coupled non-minimally with the spacetime curvature.

$$S = \int dx^4 \sqrt{-g} \left[\frac{1}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi) - \frac{1}{2}\xi R\phi^2 \right], \quad (3)$$

where ξ is the coupling constant between the scalar field ϕ and the space-time curvature R . $\xi = 0$ and $\xi = 1/6$ correspond to minimal and conformal couplings, respectively. $V(\phi)$ is the potential for the scalar field.

From the above expression, we find that an effective gravitational constant

$$G_{\text{eff}}/G = (1 - \phi^2/\phi_c^2)^{-1}, \quad (4)$$

where

$$\phi_c \equiv \frac{m_p}{\sqrt{8\pi\xi}}. \quad (5)$$

Thus it gives us a negative effective gravitational constant for a reasonable choice of $\xi > 0$, i.e., $G_{\text{eff}}/G < 0$ for $\phi > \phi_c = m_p/\sqrt{8\pi\xi}$. As mentioned in the introduction it has been shown that the anisotropic shear diverges as ϕ approaches to ϕ_c in the cases of flat and closed geometry[5–7].

When the coupling constant is in the range $0 < \xi < \frac{1}{6}$, there is another singular point

$$\hat{\phi}_c \equiv \frac{m_p}{\sqrt{8\pi\xi(1-6\xi)}} \quad (6)$$

This may be seen from the structure of the Hamiltonian constrain[5].

$$\mathcal{H} = \left(H - \frac{\phi \dot{\phi} / \phi_c^2}{1 - \phi^2 / \phi_c^2} \right)^2 + \frac{k}{a^2} - \left(\frac{4\pi \dot{\phi}^2}{3} \frac{(1 - \phi^2 / \phi_c^2)}{(1 - \phi^2 / \phi_c^2)^2} + \frac{8\pi}{3} \frac{V(\phi)}{(1 - \phi^2 / \phi_c^2)} \right) \quad (7)$$

The constraint also indicates that there is no isotropic classical solution for $\phi > \hat{\phi}_c$ in the closed or flat universe.

Only possibility to have an isotropic spacetime for $\phi > \hat{\phi}_c$ is the case where the spatial curvature is negative $k = -1$ which we will consider in detail in the following section.

3 $k = -1$ open universe with non-minimal coupling

We now consider an open isotropic universe in the presence of a nonminimally coupled scalar field. When we restrict our consideration in FRW spacetime denoted by eq. (1), the Einstein equations are found to be

$$\left(1 - \frac{\phi^2}{\phi_c^2} \right) [\dot{\alpha}^2 + k e^{-2\alpha}] = \frac{8\pi}{3} \left[\frac{\dot{\phi}^2}{2} + 6\xi \dot{\alpha} \dot{\phi} + V(\phi) \right], \quad (8)$$

where $\alpha = \ln a(t)$ and dot denotes the derivative with respect to time. The scalar field equation is found to be

$$\ddot{\phi} + 3\dot{\alpha}\dot{\phi} + V'_{\text{eff}}(\phi) = 0 \quad (9)$$

where $V_{\text{eff}}(\phi)$ is effective potential and its gradient is written as

$$V'_{\text{eff}}(\phi) = V'(\phi) + \xi R \quad (10)$$

$$= \left(1 - \frac{\phi^2}{\phi_c^2} \right)^{-1} \left[-\frac{\phi \dot{\phi}^2}{\hat{\phi}_c^2} + \left(1 - \frac{\phi^2}{\phi_c^2} \right) V'(\phi) + \frac{4\phi V(\phi)}{\phi_c^2} \right] \quad (11)$$

In the following we assume $V(\phi) = \lambda \phi^4 / 4!$ with $\lambda = 0.01$ as a typical example. We shall then numerically solve the above set of equations (8) and (9) for the case $0 < \phi_c < \phi_{\text{in}}$. We present the results of calculation with $\xi = 0.1$ ($\hat{\phi}_c \simeq 1$), $\dot{\phi}_{\text{in}} = -1.0$ and $\phi_{\text{in}} = 5, 10, 15, 20, 25$. We notice that there is upper limit for the initial value about 30 in this case otherwise one cannot satisfy eq. (8) at the initial time.

Fig.2 shows evolutions of the scalar field (upper figure) and the scale factor (lower figure). Those are scaled in Planck unit. The lines (labeled a , b , c)

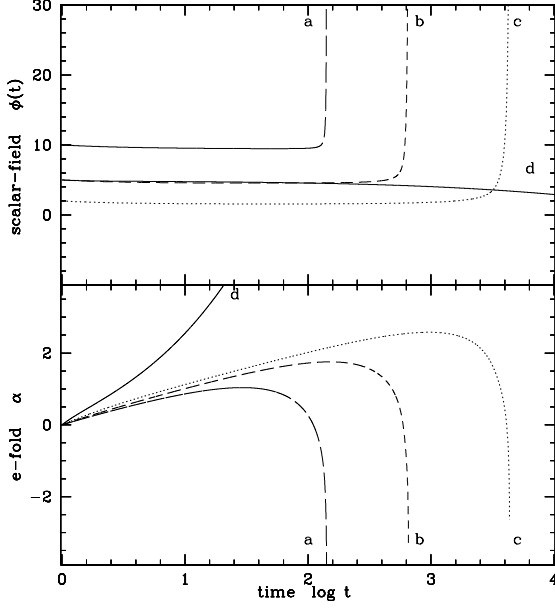


Fig. 2. The evolutions of the open isotropic universe generated by nonminimally coupled scalar field. The upper panel describes evolution of scalar field ϕ and the lower panel describes that of cosmological scale factor with the initial value of scalar field as $\phi_{\text{in}} = 10, 5, 2$,

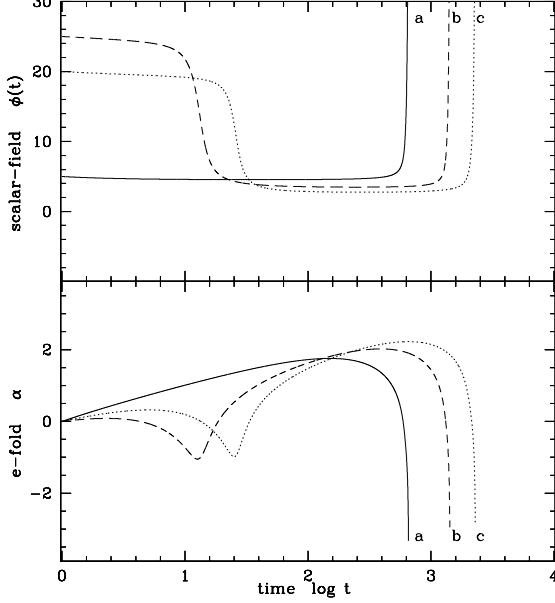


Fig. 3. As Fig. 2, but for $\phi_{\text{in}} = 5, 20, 25$.

are solutions with the initial value of scalar field as $\phi_{\text{in}} = 10, 5, 2$, respectively. These $k = -1$ open universe look like $k = -1$ open anti-de Sitter universe with negative cosmological constant $\Lambda < 0$ in Fig.1. We also draw the solid curve (labeled d) in the case of $\xi = 10^{-11}$ with $\phi_{\text{in}} = 5$ under $k=+1$ closed universe for comparison. This is a most popular case known as *chaotic inflationary universe*[3,2].

Fig.3 is the same as Fig. 2, but the lines (labeled a, b, c) are solutions with the initial value of scalar field as $\phi_{\text{in}} = 5, 20, 25$, respectively. In the case of $\phi_{\text{in}} = 5$, the scalar field ϕ seems to be blocked by barrier of the critical value $\hat{\phi}_c \simeq 1$ so as not to go through it for a long time, and then the scalar field ϕ blows up to infinitely large value. This behavior may be understood by noticing that the gradient of the effective potential diverges as the scalar field approaches to $\hat{\phi}_c$. The evolution of scale factor looks like that of de Sitter universe with $k < 0$ and $\Lambda < 0$, which necessarily come to collapse into zero ($\alpha = -\infty$) in a finite time. In the case of large initial values such as $\phi_{\text{in}} > 11$, the scalar field ϕ decreases first to near the $\hat{\phi}_c$, keep a value for a while, and blow up go to infinitely large value. Then the evolution of scale factor experiences a bounce era and goes to recollapse in the end.

4 Conclusion

We investigated the evolutionary behavior of an open isotropic universe dominated by a nonminimally coupled scalar field in the range $0 < \hat{\phi}_c < \phi_{\text{in}}$. We found by numerical analysis that the scalar field cannot cross the singular point $\hat{\phi}_c$ and instead diverges to infinity. According to the behavior of the scalar field, the expansion of the universe is turned around and is collapsing to singularity. This behavior is similar to an open de Sitter universe with a negative cosmological constant eq. (2). This could be understood by writing the expansion equation in the following form.

$$\dot{\alpha}^2 = -\frac{8\pi}{3} \frac{[\dot{\phi}^2/2 + 6\xi\dot{\alpha}\dot{\phi}\phi + V(\phi)]}{(\phi^2/\phi_c^2 - 1)} - k e^{-2\alpha} \quad (12)$$

Thus the energy density of the scalar field plays a role of a negative cosmological constant in a sense. If the cosmological constant λ_0 is not zero, the effective cosmological constant λ_{effect} is found to be

$$\lambda_{\text{effect}}(\phi, \xi) = \frac{\lambda_0}{(1 - \phi^2/\phi_c^2)} + \frac{8\pi}{3} \frac{(\dot{\phi}^2/2 + 6\xi\dot{\phi}\phi + V(\phi))}{(1 - \phi^2/\phi_c^2)}. \quad (13)$$

It is found that non positive effective cosmological constant λ_{effect} appears even if $\lambda_0 > 0$ in some range of the scalar field $\phi_{\text{in}} > \phi_c$. Thus it might give us a possible mechanism to reduce the cosmological constant to a small value.

There are also other solutions which show a rebounding behavior before the eventual collapse to the singularity. It seems to us that this kind of behavior has not been known before. Just after the bounce the expansion is accelerated.

Then it might be possible to have a new open inflationary scenario by adjusting parameters (initial value of the scalar field and the coupling constant).

It also interesting to consider how anisotropic shear behaves in the singular point in our open universe. Our preliminary result show that the anisotropy diverges also as one approaches to ϕ_c also in the open universe. This will be published elsewhere.

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